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Large Volume Limit of the Distribution of Characteristic Exponents in Turbulence

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Abstract. For spatially extended conservative or dissipative physical systems, it appears natural that a density of characteristic exponents per unit volume should exist when the volume tends to infinity. In the case of a turbulent viscous fluid, however, this simple idea is complicated by the phenomenon of intermittency. In the present paper we obtain rigorous upper bounds on the distribution of characteristic exponents in terms of dissipation. These bounds have a reasonable large volume behavior. For two-dimensional fluids a particularly striking result is obtained: the total information creation is bounded above by a fixed multiple of the total energy dissipation (at fixed viscosity). The distribution of characteristic exponents is estimated in an intermittent model of turbulence (see [7]), and it is found that a change of behavior occurs at the value $D = 2.6$ of the self-similarity dimension.

To Freeman J. Dyson, Res Jost, and Arthur S. Wightman

The relation between physics – real physics – and mathematics – real mathematics – has not been an easy one in the last thirty years. It took vision to see that this relation is possible and fruitful now as it was in the times of Archimedes, Newton, and Einstein. Res Jost in Zürich, Freeman Dyson and Arthur Wightman in Princeton had that vision, and made many others share it.

0. Introduction

The time evolutions of relevance to physics often define nonlinear differentiable dynamical systems of the form

$$\frac{dx}{dt} = F(x), \tag{0.1}$$

where x varies in a vector space or manifold which may have infinite dimension. Let the initial condition x_0 for (0.1) be replaced by $x_0 + \delta x_0$; if the following limit exists

$$\mu(x_0, \delta x_0) = \lim_{t \to \infty} \frac{1}{t} \log \|\delta x_t\|, \tag{0.2}$$

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