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Weak Convergence of a Random Walk in a Random Environment

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Abstract. Let $\pi_i(x)$, $i=1,\ldots,d$, $x\in Z^d$, satisfy $\pi_i(x)\geqq\alpha>0$, and $\pi_1(x)+\ldots+\pi_d(x)=1$. Define a Markov chain on Z^d by specifying that a particle at x takes a jump of $+1$ in the i^{th} direction with probability $\frac{1}{2}\pi_i(x)$ and a jump of -1 in the i^{th} direction with probability $\frac{1}{2}\pi_i(x)$. If the $\pi_i(x)$ are chosen from a stationary, ergodic distribution, then for almost all π the corresponding chain converges weakly to a Brownian motion.

1. Introduction

Let Z^d be the integer lattice and let e_i, $i=1,\ldots,d$, denote the unit vector whose i^{th} component is equal to 1. Let

$$S=\{(p_1,\ldots,p_d)\in\mathbb{R}^d: p_i\geqq 0, p_1+\ldots+p_d=1\},$$

and suppose we have a function $\pi:Z^d\to S$. Then a Markov chain $X_\pi(j)$ on Z^d is generated with transition probability

$$P\{X_\pi(j+1)=x\pm e_i\,|\,X_\pi(j)=x\}=\tfrac{1}{2}\pi_i(x), \tag{1.1}$$

and generator

$$L_\pi g(x)=\sum_{i=1}^{d}\tfrac{1}{2}\pi_i(x)\{g(x+e_i)+g(x-e_i)\}.$$

If the function π is chosen from some probability distribution on S, this gives an example of a random walk in a random environment.

For any π, we can consider the limiting distribution of the process X_π satisfying $X_\pi(0)=0$ and (1.1). Let $\alpha>0$ and set

$$S^\alpha=\{(p_1,\ldots,p_d)\in S: p_i\geqq\alpha\},$$

and let C^α be the set of functions $\pi:Z^d\to S^\alpha$. The main result of this paper is:

Theorem 1. *Let μ be a stationary ergodic measure on C^α. Then there exists $b\in S^\alpha$ such*

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