Commun. Math. Phys. 87, 37–63 (1982)

Communications in
**Mathematical
Physics**

Supergravity, Complex Geometry and G-Structures

A. S. Schwarz

Moscow Physical Engineering Institute, Kashirskoe Shosse 1, Moscow M-409, USSR

Abstract. The geometry of supergravity is studied. New formulations of supergravity are given. The equivalence of different approaches to supergravity is analyzed.

Introduction

The subject of the present work is the geometrical aspect of supergravity. (We mean the $N=1$ supergravity everywhere.) There are various geometrical approaches to the supergravity theory. The most elegent one is due to V. Ogievetsky and E. Sokatchev [1], [2]. In that approach the role of the field is played by a (4,4)-dimensional surface in $\mathbb{C}^{4,2}$ (the complex superspace of the complex dimension (4,2)). Another approach, that by Wess and Zumino, is based upon the concept of the frame[1] fields in the (4,4)-dimensional superspace; the frame field in this approach is determined up to a transformation belonging to the Lorentz group. The equivalence between the Ogievetsky–Sokatchev approach and the Wess–Zumino approach was established previously [2] by means of a rather cumbersome consideration and its geometrical background is not always quite clear.

A purpose of the present investigation is to analyze the internal geometry of (4,4)-dimensional surfaces embedded into the space $\mathbb{C}^{4,2}$. The analysis results first of all in a simpler construction of the action functional than the Ogievetsky–Sokatchev approach [2, 3]. Namely, it was found that the action is simply expressed in terms of the so-called Levi form for the surface. Moreover, the analysis of the geometry in the surface provides manifest way to establish a correspondence between the Ogievetsky–Sokatchev and Wess–Zumino methods.

Our construction is based on the theory of G-structures[4]. The meaning of this statement is that the surface geometry is given by a frame field which is determined not up to a local Lorentz rotation, as it was in the Wess–Zumino approach, but by the frame field determined up to a transformation belonging to an arbitrary linear group G. In other words, two frames $\tilde{E}_A^M(x)$ and $E_A^M(x)$ are considered as corresponding to the same geometry, if $\tilde{E}_A^M(x)=g_A^B(x)E_B^M(x)$, where $g_A^B(x)$ is a function taking its values in the group G. The frame fields determining the

1 We prefer the term "frame" instead of "supertetrade" and "Vielbein" used in the physics literature

0010-3616/82/0087/0037/$05.40