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$(\mathrm{Higgs})_{2,3}$ Quantum Fields in a Finite Volume

II. An Upper Bound*

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Abstract. This is the second part of the paper entitled, "$(\mathrm{Higgs})_{2,3}$ Quantum Fields in a Finite Volume." The proof of an upper bound for vacuum energy is completed with the exception of some technical estimates.

1. Introduction

This paper is a second part of the paper [1] and contains the second, more important part of the proof of the theorem formulated there. Let us recall the basic definitions and the theorem. We consider two spaces of field configurations on the torus

$$T_\varepsilon = \{x \in \varepsilon Z^d : -L_\mu \leqq x_\mu < L_\mu, \mu = 1, \ldots, d\}:$$

scalar fields and vector fields. Scalar field configurations are the functions $\phi : T_\varepsilon \to R^N$. Vector field configurations are the functions $A : T_\varepsilon \to R^d$ identified with the functions $A : T_\varepsilon^* \to R$ by the equality: $A_{\langle x, x+\varepsilon e_\mu \rangle} = A_\mu(x)$. Of course the periodic boundary conditions are understood here if the torus is identified with the subset of εZ^d. We consider the action

$$\begin{aligned} S^\varepsilon(A,\phi) = \tfrac{1}{2} \sum_{b \subset T_\varepsilon} \varepsilon^d |(D_A^\varepsilon \phi)(b)|^2 + \sum_{x \in T_\varepsilon} \varepsilon^d (\tfrac{1}{2} m_0^2 |\phi(x)|^2 \\ + \lambda |\phi(x)|^4) + \tfrac{1}{2} \sum_{b \subset T_\varepsilon} \varepsilon^d |(\partial^\varepsilon A)(b)|^2 + \tfrac{1}{2} \mu_0^2 \sum_{x \in T_\varepsilon} \varepsilon^d |A(x)|^2 - E, \end{aligned} \tag{1.1}$$

where $m_0^2 = m^2 + \delta m^2$, $m^2 > 0$ and δm^2 is the mass renormalization counterterm, $\mu_0^2 > 0$, $\lambda > 0$ and $E = E_0 + E_1$, E_0 is the normalization factor and E_1 is the renormalization counterterm of vacuum energy. The counterterms δm^2 and E_1 are defined with the help of perturbation expansions. The more detailed description of (1.1) is given in the first part [1, Chap. 1].

The partition function is defined as usual,

$$Z^\varepsilon = \int dA \int d\phi \exp(-S^\varepsilon(A,\phi)), \tag{1.2}$$

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