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Asymptotic Behaviour of the Boltzmann Equation with Infinite Range Forces

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Abstract. We have previously obtained existence results for the space-homogeneous, non-linear Boltzmann equation for a class of encounters with infinite range, including inverse k^{th} power molecules with $k > 3$. In the present paper those solutions are proved to converge in weak L^1-sense for $k \geqq 5$ to Maxwellian distributions when $t \to \infty$. Also the higher moments converge to those of the relevant Maxwellian. The method of proof relies on non-standard techniques.

1. Preliminaries

Consider the space-homogeneous, non-linear Boltzmann equation in the absence of exterior forces

$$D_t f(t, v) = Qf(t, v) \qquad (t > 0, v \in R^3) \tag{1}$$

with Cauchy data

$$f(0, v) = f_0(v) \geqq 0, \quad (v \in R^3).$$

Here $Q = Q_k$ denotes the collision operator

$$Qf(v_1) = Q_w f(v_1) = \int_{R^3 \times B^2} [f \otimes f(J_u(v_1, v_2)) - f \otimes f(v_1, v_2)] w(v_1, v_2, u) dv_2 du, \tag{2}$$

and

$$f \otimes g(v_1, v_2) = f(v_1) g(v_2).$$

In the cut-off case the impact parameter u in R^2 is restricted to a disc

$$B^2(r) = \{u \in R^2; \quad |u| \leqq r\}.$$

In the present paper, however, with intermolecular forces of infinite range, we consider

$$B^2 = B^2(\infty) = R^2.$$

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