Commun. Math. Phys. 86, 363–373 (1982)

Communications in
Mathematical
Physics

Local Stability and Hydrodynamical Limit of Spitzer's One Dimensional Lattice Model

J. Fritz

Mathematical Institute, H-1053 Budapest, Reáltanoda u. 13, Hungary

Abstract. An infinite system of ordinary differential equations is considered, the right hand side is just the negative gradient of potential energy of a one-dimensional system of unbounded spins interacting by a symmetric and convex pair potential. Constant configurations are stationary points and the mean spin is conserved. It is shown that each of these stationary points has its own domain of attraction, the initial distribution need not be translation invariant. As a consequence we obtain that the mean spin satisfies the heat equation in the hydrodynamical limit.

0. Introduction

One of the most striking difficulties in the study of temporal evolution of large physical systems is certainly the existence of whole families of stationary states. Although degeneracy of the stationary state is usually associated to conservation principles, relaxation to equilibrium is as yet poorly understood. Perhaps the simplest but not exactly solvable model of this kind is the following gradient dynamics of one-dimensional systems of unbounded spins. Let $\mathbb{R}$ denote the real line, let $\mathbb{Z}$ be the set of integers, and suppose that we are given a continuously differentiable, symmetric and convex function $U:\mathbb{R}\to[0,+\infty)$. Elements of the product space $\mathbb{R}^{\mathbb{Z}}$ are represented as doubly infinite sequences $\omega=(\omega_k)_{k\notin\mathbb{Z}}$, i.e. ω_k denotes the k^{th} co-ordinate of $\omega\in\mathbb{R}^{\mathbb{Z}}$. The purpose of this paper is to investigate asymptotic behaviour of solutions to the Cauchy problem for

$$\frac{d\omega_k(t)}{dt}=-U'(\omega_k(t)-\omega_{k-1}(t))-U'(\omega_k(t)-\omega_{k+1}(t)), \tag{0.1}$$

where $t\geqq 0$, $k\in\mathbb{Z}$ and U' denotes the derivative of U. Of course, (0.1) will be considered only in an appropriately chosen subset of $\mathbb{R}^{\mathbb{Z}}$. Let us remark that symmetry of U implies a conservation law for the mean spin, and configurations of type $\omega_k=\mu+\lambda k$ are stationary points of (0.1).

0010-3616/82/0086/0363/$02.20