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Decay of Correlations for Infinite Range Interactions in Unbounded Spin Systems*

Camillo Cammarota

Istituto di Matematica, Università di Roma, C.N.R. (G.N.F.M.), I-00185 Roma, Italy

Abstract. In unbounded spin systems at high temperature with two-body potential we prove, using the associated polymer model, that the two-point truncated correlation function decays exponentially (respectively with a power law) if the potential decays exponentially (respectively with a power law). We also give a new proof of the convergence of the Mayer series for the general polymer model.

1. Definitions and Results

In the finite subset Λ of $\mathbb{Z}^d$ we consider the collection of random variables $S_\Lambda = \{S_x \in \mathbb{R}^\nu, x \in \Lambda\}$ distributed with the Gibbs probability measure, i.e.,

$$Z_\Lambda^{-1} e^{-\beta \sum_{\substack{X \subset \Lambda \\ |X| \geqq 2}} \Phi_X(S_X)} W_\Lambda(dS_\Lambda), \tag{1.1}$$

where Φ is a given many-body potential,

$$W_\Lambda(dS_\Lambda) = \prod_{x \in \Lambda} W_x(dS_x),$$

$$W_x(dS_x) = \left(\int \mu_x(dS_x) \exp - \beta\Phi_x(S_x)\right)^{-1} (\exp - \beta\Phi_x(S_x)) \mu_x(dS_x), \tag{1.2}$$

where μ_x is the *a priori* single spin distribution and β is the inverse temperature, Z_Λ is the partition function and $|X|$ is the number of points of X.

The finite volume correlation functions are

$$\varrho_\Lambda(S_X) = Z_\Lambda^{-1} \int W_{\Lambda\setminus X}(dS_{\Lambda\setminus X}) \exp - \beta \sum_{\substack{X \subset \Lambda \\ |X| \geqq 2}} \Phi_X(S_X). \tag{1.3}$$

Our first result is the following theorem:

Theorem 1. *Let Φ be a two-body potential such that*

$$|\Phi_{xy}(S_x S_y)| \leqq e^{-\delta(x,y)} J(x,y) v_x(S_x) v_y(S_y), \tag{1.4}$$

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