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Boundary Regularity for the Navier-Stokes Equations in a Half-Space*

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Abstract. Weak solutions to the nonstationary Navier-Stokes equations in a half-space are locally bounded at the boundary except for a closed set with finite one-dimensional Hausdorff measure.

1. Introduction

The purpose of this paper is to show that weak solutions u to the nonstationary Navier-Stokes equations in a half-space satisfy a regularity condition at the boundary. This regularity condition says that, except for a closed singular set whose one-dimensional Hausdorff measure is finite, u is locally bounded at the boundary of the half-space. The precise statement of this result is contained in Theorem 1.1 below.

In [1] it was proved that, at least in the case of a bounded domain, the interior singularities of the vorticity of u are concentrated in a locally closed set whose one-dimensional Hausdorff measure is finite. The vorticity of u can be replaced by u in the preceding statement. Theorem 1.1 extends that research to the boundary of the domain. It is interesting to note that the dimension does not jump up when we reach the boundary.

Our half-space will be $U=\{(x_1,x_2,x_3)\in R^3 : x_3>0\}$, its boundary will be denoted $B(U)$, and the set of positive times will be $R^+=\{t : t>0\}$. The weak solution u is a function which is defined on $U\times R^+$. It is convenient to extend u by zero, so that it becomes a function on $R^3\times R^+$. The spatial gradient of u (where we do not include the partial derivative with respect to time) will be written Du.

Theorem 1.1. *If $\bar{w} : R^3\to R^3$ is an L^2 function, $\bar{w}(x)=0$ when $x\notin U$, and* $\operatorname{div}(\bar{w})=0$ *then there exist $u : R^3\times R^+\to R^3$ and $S\subset B(U)\times[0,\infty)$ such that the following conditions hold:*

(1) *$u(x,t)=0$ when $x\notin U$; $Du\in L^2$.*

(2) *u is a weak solution to the nonstationary Navier-Stokes equations of incompressible fluid flow in U with viscosity $=1$ and initial condition $\bar{w}$.*

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