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# On the Vortex Flow in Bounded Domains

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**Abstract.** We consider the motion of $N$ vortices in bounded domains in $\mathbb{R}^2$. We prove that the set of initial positions which lead to a collapse of two or more vortices has Lebesgue measure zero. We extend this result to the stochastic motion of the vortices, where the stochasticity comes from a Wiener-noise term, which is added to the deterministic equation of motion.

## 1. Introduction

A system of $N$ vortices ($x_i \in D$ is the position of the $i^{\text{th}}$ vortex and $\gamma_i \in \mathbb{R}$ its vorticity) in a domain $D \subseteqq \mathbb{R}^2$ satisfies the following equations of motion

$$\dot{x}_i(t) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \gamma_j K(x_i(t), x_j(t)), \qquad i = 1, \ldots, N, \tag{1.1}$$

with

$$K(x, y) = (\nabla_x^{\perp} g)(x, y); \qquad x, y \in D, \tag{1.2}$$

$g$ being the fundamental solution of the Poisson equation with the appropriate boundary conditions, $\nabla^{\perp} = (\partial_2, -\partial_1)$, and $\partial_k$, $k = 1, 2$ is the partial derivative with respect to the $k^{\text{th}}$ component.

The evolution equation (1.1) is equivalent to the Euler equation (except for an infinity constant due to the self-energy factor) and describes the dynamics of an incompressible fluid in which the vorticity is sharply concentrated around the points $x_i$. This model was introduced by Kirchhoff [1]. See also [2] for a precise connection with the Euler equation.

The following three choices of the domain $D$ are of interest for physics.

(i) $D = \mathbb{R}^2$, i.e. $g(x, y) = -\dfrac{1}{2\pi} \ln(|x - y|)$.

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