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On the Inverse Problem in Statistical Mechanics

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Abstract. For quantum spin systems it is known that for a suitable space of potentials the equilibrium states are W^*-dense in the set of all translation invariant states. The problem discussed in this paper is how to recognize such equilibrium states and how to find the corresponding potential. A necessary and sufficient condition for a state to be an equilibrium state for some potential is given in Sect. 3.

1. Introduction

The problem of equilibrium statistical mechanics is to determine the equilibrium state of a system for given temperature and given interaction (and possibly other parameters such as chemical potential, external field, etc.). Except for the most trivial models this is a formidable problem. Only in exceptional cases some correlation function can be calculated rigorously. In most cases one has to rely on approximation methods, such as perturbation theory. However, in the neighborhood of critical potentials even such approximation methods fail, or, if they give satisfactory results, it is not quite understood why they work.

Notwithstanding these computational difficulties the problem is well-defined, at least in the case of quantum lattice systems. The interaction between spins is determined by a potential. For each potential in a certain Banach space of potentials there is at least one equilibrium state. In case there is more than one equilibrium state for a given potential, this set is shown to be a simplex, so that each such state can be decomposed uniquely in terms of the set of extreme equilibrium states (the pure phases). The set of all equilibrium states for all possible potentials is W^*-dense in the set of all translation invariant states [1].

The inverse problem in statistical mechanics is to determine the potential for which a given state is in equilibrium. This problem has two aspects, an existence aspect and a computational aspect. As not every state is an equilibrium state for some potential, it is of interest to have sufficient conditions for a state to be an equilibrium state for some potential. Another related problem is how to determine this potential even if one knows that the state is in equilibrium. In other words, can

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