Commun. Math. Phys. 84, 557–578 (1982)

Communications in Mathematical Physics

Construction of Non-Gaussian Self-Similar Random Fields with Hierarchical Structure

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Abstract. In the present work we construct non-Gaussian self-similar random fields with hierarchical structure. The construction is based on non-Gaussian solutions of the main nonlinear equation of the hierarchical models theory. The existence of such solutions was proved originally by Sinai and the author and later by another method by Collet and Eckmann. Next we establish the uniqueness of a Gibbs state for the constructed self-similar field. Finally for a class of hierarchical models we prove the convergence of renormalization transformations of a random field at the critical point to the self-similar field.

1. Definitions

Let $r\in\mathbb{Z}$, $r\geqq 2$, and $\xi_0>\xi_1>\xi_2>\dots$ be a decreasing sequence of partitions of a countable set V satisfying the following conditions:

(i) ξ_0 is the partition of V into separate points,

(ii) any element of the partition ξ_n consists of r elements of the partition ξ_{n-1}, $n=1,2,\dots$,

(iii) for any two points $i,j\in V$ a number n exists such that i,j belong to the same element of the partition ξ_n.

Such a sequence of partitions is called a hierarchical structure in the set V (see [1]). Let us denote $n(i,j)$ the least number n such that i,j belong to the same element of the partition ξ_n. The quantity

$$d(i,j)=\begin{cases}0 & \text{if} \quad i=j\\ r^{n(i,j)} & \text{if} \quad i\neq j\end{cases}$$

defines a metrics in a set V with hierarchical structure.

A map $V\to V'$ is called an isomorphism of hierarchical structures if it preserves the structure of the partitions. One can see easily that for a given r any two hierarchical structures are isomorphic. We shall consider two realizations of the

0010-3616/82/0084/0557/$04.40