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On Edwards' Model for Polymer Chains
III. Borel Summability

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Abstract. The basic existence theory for Edwards' model of long polymer chains is completed.

1. Introduction

In [1] we proved an existence theorem for a probability measure on continuous paths in space, proposed by Edwards [2] as a stochastic model for the geometric properties of long polymer chains. This theorem was limited to sufficiently small values of the coupling constant g, and, as noted in the introduction to [3], this restriction is highly unsatisfactory, since the most interesting question concerning the Edwards model is a question about its asymptotic behavior in the limit $g \to \infty$. In the present paper we show that the polymer measure $\nu(g)$ is well defined for all $g \geqq 0$. In addition we show that the (renormalised) perturbation series for moments of $\nu(g)$, although (presumably) divergent, determine the moments as their Borel sums. This result is of interest because it shows that all information about the model is implicitly contained in the perturbation series, and hence guarantees the uniqueness of our construction of the model. In a final section we confirm the expectation of Symanzik [4], that the measure $\nu(g)$, $g > 0$, and the Wiener measure $\mu = \nu(0)$ are mutually singular, by showing that $\nu(g_1)$ and $\nu(g_2)$ are disjoint for any g_1, g_2 with $0 \leqq g_1 < g_2$.

Throughout this paper we rely heavily on the results of [1]. In particular the notation of [1] will be used without further explanation.

2. Borel Summability

We adopt the notation and hypotheses of Theorem 1 [1], and consider, for a given localised random variable $R \in L^q(\Omega, G(m), \mu)$, for some $m \geqq 0$, $q > 1$, the expectation

$$\begin{aligned} E_{\nu(g)}[R] &= E[R f_m(g)] \\ &= \lim_{n \to \infty} \frac{E[R \exp[-gS(n)]]}{E[\exp[-gS(n)]]} \\ &= F(R,g)[F(1,g)]^{-1}, \end{aligned} \tag{1}$$

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