Commun. Math. Phys. 83, 493–515 (1982)

Communications in Mathematical Physics

First-Order Phase Transitions in Large Entropy Lattice Models

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Abstract. We show the existence of a first-order phase transition in the ν-dimensional Potts model for $\nu \geqq 2$, when the number of states of a single spin is big enough. Low-temperature pure phases are proved to survive up to the critical temperature. Also the existence of a first-order transition in the ν-dimensional Potts gauge model, $\nu \geqq 3$, is obtained if the underlying gauge group is finite but large.

1. Introduction

In [1], Potts introduced a generalization of the Ising model by enlarging the number of values taken by spins residing on lattice sites from 2 to an arbitrary q, and considering a nearest neighbour interaction that is, up to a factor, -1 when the neighbouring spins are alike and 0 when they differ. The problem of the description of the phase structure of the Potts model was later studied by many authors using different methods. We refer the reader to the recent review by Wu [2] for a description of present knowledge about the Potts model. Existent rigorous results are confined mostly to the Potts model on a two-dimensional lattice and are based on its equivalence to an ice-rule vertex model. In particular it seems to have been proven [3, 4] that the two-dimensional Potts model exhibits a first-order transition in temperature for $q > 4$. Essentially nothing was proved for three and higher dimensional lattices though it is believed that the phase transition is first order for $q \geqq 4$ (or even 3) in three dimensions and $q \geqq 3$ in higher dimensions. The conjecture that for large q, the Potts model undergoes a first-order transition is also supported by Pearce and Griffiths [5] who prove that for a lattice of any dimension the mean field approximation of a free energy of the model is exact in the limit $q \to \infty$. At the same time, the suitably rescaled limiting mean field free energy can be easily computed and it turns out that its behaviour is

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0010-3616/82/0083/0493/$04.60