Commun. Math. Phys. 83, 151–170 (1982)

Communications in
Mathematical
Physics
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Perturbation Theory for Shape Resonances and Large Barrier Potentials

Mark S. Ashbaugh[1*] and Evans M. Harrell II[2**]

[1] Department of Mathematics, University of Tennessee, Knoxville, TN 37916, USA
[2] Department of Mathematics, The Johns Hopkins University, Baltimore, MD 21218, USA

Abstract. We develop a systematic perturbation and resonance theory for the one-dimensional Schrödinger equation of the form

$$(-d^2/dx^2 + U(x) + \lambda V(x) - E)\psi(x) = 0, \quad 0 \leqq x < \infty,$$

where the barrier potential $V(x)$ is supported only where $x \geqq 1$ and is non-negative there, and λ is a real parameter tending to infinity. We prove that every $\lambda = \infty$ eigenvalue turns into a resonance or an eigenvalue for finite λ.

1. Introduction

One of the standard problems treated in elementary quantum mechanics textbooks is the decay of a nucleus via alpha emission. The discussion is customarily based on a simplified model wherein the alpha particle is acted upon by a spherically symmetric potential comprising a short-range negative piece from the attraction between nucleons and a long-range repulsive piece from the electric interaction between protons. The large potential barrier has two effects, viz., that it confines the alpha particle for long periods until it escapes by tunneling, and that it alters the energy levels of the bound or quasi-bound states. A representative discussion is to be found in [1].

This paper addresses the two principal mathematical problems connected with this model:

1. To develop a perturbation series for the bound states and resonance energies in the limit as the barrier size is elevated to infinity. This does not appear to have been done systematically before, probably owing to the very singular sort of limit as $\lambda \to \infty$. It will turn out that the series involves fractional powers of $1/\lambda$, λ being the variable parametrizing the height of the barrier; in particular, when $V(x) \sim$

* Partially supported by USNSF grant MCS 7801885 and a National Science Foundation Graduate Fellowship

** Supported by USNSF grant MCS 7926408

0010-3616/82/0083/0151/$04.00