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Renormalization Group Study of a Critical Lattice Model

I. Convergence to the Line of Fixed Points

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Abstract. We start a nonperturbative study of the Wilson-Kadanoff renormalization group (RG) in weakly coupled massless lattice models. Nonlocal hierarchical models are introduced to mimic the infrared behaviour of the $\frac{1}{2}(\nabla\phi)^2+\lambda(\nabla\phi)^4$ model and the like. The RG is shown to drive these to the line of fixed points corresponding to the massless $\frac{1}{2}c_\infty(\lambda)(\nabla\phi)^2$ models.

1. Introduction

The present paper is a (self-contained) continuation of the program started by [11]. We aim at a rigorous theory of weakly coupled massless lattice models, a counterpart of the high and low temperature cluster expansions developed for the massive case. Our approach parallels other recent attempts of rigorously studying massless models like $\lambda(\nabla\phi)^4$, the dipole gas, the low temperature Coulomb gas or plane rotator [3–5, 7–9]. It is centered around the idea of the renormalization group.

In [11] we have exhibited the block spin structure of the free Gaussian model $\frac{1}{2}(\nabla\phi)^2$ in $d\geqq 2$ dimensions by writing

$$\nabla\phi_x=(\nabla QZ^0)_x+3^{-\frac{d}{2}}(\nabla\mathscr{A}_1QZ^1)_{\frac{x}{3}}+\ldots+3^{-\frac{dk}{2}}(\nabla\mathscr{A}_kQZ^k)_{\frac{x}{3^k}}+\ldots, \tag{1}$$

where the kernel $(\nabla\mathscr{A}_kQ)_{zy}$, $z\in 3^{-k}\mathbb{Z}^d$, $y\in\mathbb{Z}^d$, is concentrated around $z\sim y$ and decays exponentially for $|z-y|\to\infty$ uniformly in k. The Gaussian fluctuation fields Z^k are independent for different k and their covariances possess an exponential decay uniform in k.

Our hierarchical model is patterned on this structure. Here are the main simplifications we introduce when constructing it:

1. the number of random variables Z_y^k, $y\in\mathbb{Z}^d$, is reduced to one $Z_{y_0}^k$ for each block of 3^d sites centered around $3y_0$ (in the original model there were 3^d-1 variables),

2. all Z^k fields are taken as equally distributed,

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