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Adiabatic Theorem and Spectral Concentration

I. Arbitrary Order Spectral Concentration for the Stark Effect in Atomic Physics

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Abstract. The spectral concentration of arbitrary order for the Stark effect is proved to exist for a large class of Hamiltonians appearing in nonrelativistic and relativistic quantum mechanics. The results are consequences of an abstract result about the spectral concentration for self-adjoint operators. A general form of the adiabatic theorem of quantum mechanics, generalizing an earlier result of the author as well as some results by Lenard, is also proved.

1. Introduction

This is the first in a series of papers devoted to the study of some asymptotic phenomena appearing in the spectral theory of linear operators and in the theory of evolution equations in Hilbert (or, more generally, Banach) spaces. Common to all the papers in the series will be the method employed which is, we believe, a new and rather general way of performing the asymptotic expansions. In a less abstract form, the basic ideas of our method have already appeared in [1–3].

In this paper we shall prove two results. The first one (Theorem 1) gives the existence of asymptotically invariant subspaces (see Sect. 2 for precise definitions) for a class of families, H_ε, $\varepsilon \geqq 0$, of self-adjoint operators in Hilbert spaces. For finite dimensional asymptotically invariant subspaces our result has a close relation to the abstract theory of spectral concentration as developed in [4, 5] (see also [6, Chap. VIII, Sect. 5] and [7, Chap. XII]). The second result (Theorem 2) is an adiabatic theorem "to an arbitrary order" generalizing a recent result of the author [1] as well as some results of Lenard [8].

As an application of Theorem 1 we shall prove the existence of spectral concentration of arbitrary order for the Stark Hamiltonians of atomic physics: atoms and molecules, impurity states in solids, relativistic hydrogen atom etc., as well as for Hamiltonians describing barrier penetration phenomena.

Concerning the Stark effect in atomic physics, some remarks are in order. In the framework of the abstract theory of spectral concentration, Riddell [4] and (in

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