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A C*-Algebra Approach to the Schwinger Model

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Abstract. If cutoffs are introduced then existing results in the literature show that the Schwinger model is dynamically equivalent to a boson model with quadratic Hamiltonian. However, the process of quantising the Schwinger model destroys local gauge invariance. Gauge invariance is restored by the addition of a counterterm, which may be seen as a finite renormalisation, whereupon the Schwinger model becomes dynamically equivalent to a linear boson gauge theory. This linear model is exactly soluble. We find that different treatments of the supplementary (i.e. Lorentz) condition lead to boson models with rather different properties. We choose one model and construct, from the gauge invariant subalgebra, a class of inequivalent charge sectors. We construct sectors which coincide with those found by Lowenstein and Swieca for the Schwinger model. A reconstruction of the Hilbert space on which the Schwinger model exists is described and fermion operators on this space are defined.

1. Introduction

In a series of previous papers [1–6] we have developed a definite metric quantisation procedure for linear boson field theories which contain massless particles. In the case of the free electromagnetic field this rigorises Fermi's original quantisation procedure [7] and among other things provides a natural framework for a discussion of gauge transformations [3], [4]. It has an advantage over the indefinite metric approach in that standard C^* algebra and Hilbert space methods may be applied.

To see how these ideas might work in a fully interacting theory we decided to attempt an analysis of the Schwinger model [8] [$(\mathrm{QED})_2$ with massless Fermions] and $(\mathrm{QED})_2$ itself. In this paper we discuss only the Schwinger model from our viewpoint. There is a fortunate accident which occurs as a consequence of the masslessness of the Fermions in the Schwinger model. Namely the model may be "bosonised" in the sense that, as far as the observables of the model are concerned, the dynamics can be expressed solely in terms of currents without reference to the fermion fields themselves. This possibility does not seem to be

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