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Possible New Strange Attractors With Spiral Structure

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Abstract. We define a class of three-dimensional differential equations which might present strange attractors of a new kind. This is illustrated by numerical observations on an explicit example.

I. Introduction

These last years a lot of efforts have been devoted to the study of chaotic behaviours which may arise in some dynamical systems. The relevance of these works to understand the transition to turbulence in physical and chemical experiments seems now well established [1]. A mathematical theory of Axiom A "strange attractors" has been fairly achieved and provides a satisfying description of some systems which do have complicated asymptotic behaviour [2, 3]. Nevertheless this theory does not allow us to understand the stochasticity generated in numerical investigations of diffeomorphisms and differential equations given by explicit algebraic expressions. Indeed most of these numerical studies deal with generalizations of the Henon mapping [4, 5] or its suspensions [6]. The existence of chaotic behaviour can be proved in these cases [7–10] but, according to [11], the very existence of strange attractors is far from being settled since homoclinic tangencies [12] arise quite naturally with one-parameter families of such systems[1]. Perhaps a special status should be attributed to the Lorenz system [14] which could fall in the class of topological models defined by Guckenheimer [15] and which posses effective strange attractors [16, 17].

In this paper we propose a construction of what might reveal to be a new kind of strange attractors. This construction is illustrated by numerical observations on an explicit example of differential equations describing a forced oscillator [18].

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1 Recently Misiurewicz [13] proved the existence of strange attractors for the homeomorphism $(x, y) \to (1 - a|x| + by, x)$, under some conditions on the parameters *a and b*

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