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# Stability and Isolation Phenomena for Yang-Mills Fields

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**Abstract.** In this article a series of results concerning Yang-Mills fields over the euclidean sphere and other locally homogeneous spaces are proved using differential geometric methods. One of our main results is to prove that any weakly stable Yang-Mills field over $S^4$ with group $G = SU_2, SU_3$ or $U_2$ is either self-dual or anti-self-dual. The analogous statement for $SO_4$-bundles is also proved. The other main series of results concerns gap-phenomena for Yang-Mills fields. As a consequence of the non-linearity of the Yang-Mills equations, we can give explicit $C^0$-neighbourhoods of the minimal Yang-Mills fields which contain no other Yang-Mills fields. In this part of the study the nature of the group $G$ does not matter, neither is the dimension of the base manifold constrained to be four.

## 1. Introduction and Statement of Results

The purpose of this article is to prove a series of results concerning Yang-Mills fields over the euclidean sphere and other locally homogeneous spaces by using differential geometric methods. Many of these results were announced in [7].

Our basic set-up is the following. We consider a compact riemannian manifold $M$ and a principal $G$-bundle $P$ over $M$ where $G$ is a compact Lie group. On the space $\mathscr{C}_P$ of connections on $G$ we consider the *Yang-Mills functional*

$$\mathscr{YM}(\nabla) = \frac{1}{2} \int_M \|R^\nabla\|^2 ,$$

where $R^\nabla$ is the curvature of the connection $\nabla$ in $\mathscr{C}_P$ and where the norm is defined in terms of the riemannian metric on $M$ and a fixed $\mathrm{Ad}_G$-invariant scalar product on the Lie algebra $\mathfrak{g}$ of $G$.

Critical points of the smooth function $\mathscr{YM} : \mathscr{C}_P \to \mathbb{R}$ are precisely those connections whose curvature tensors are "harmonic". These critical points are

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