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Critical Point Dominance in One Dimension

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Abstract. The renormalized, dimensionless 4-point coupling constant of scalar one dimensional field theories is maximized uniquely by the "critical point theories" (obtainable as the scaling limit of ϕ^4 models). The renormalized coupling constant of certain scalar one dimensional lattice field theories is maximized uniquely (for fixed correlation length) by the corresponding spin-1/2 model.

1. Introduction

For a scalar, Euclidean field, $\phi(x)$, $x \in \mathbb{R}^d$, with truncated Schwinger (Ursell) functions $U_n(x_1, \ldots, x_n)$, and physical mass $m > 0$, one definition of the renormalized, dimensionless coupling constant g, which is particularly appropriate for ϕ^4 models (see [6]) is

$$g = m^d B / A^2 , \tag{1}$$

where

$$A = \lim_{L \to \infty} L^{-d} \int_{(C_L)^2} U_2(x_1, x_2) dx_1 dx_2 = \int_{\mathbb{R}^d} U_2(0, x) dx , \tag{2}$$

$$\begin{aligned} B &= \lim_{L \to \infty} L^{-d} \int_{(C_L)^4} [-U_4(x_1, x_2, x_3, x_4)] dx_1 dx_2 dx_3 dx_4 \\ &= \int_{(\mathbb{R}^d)^3} [-U_4(0, y_1, y_2, y_3)] dy_1 dy_2 dy_3 , \end{aligned} \tag{3}$$

and C_L denotes the cube, $([-L/2, L/2])^d$, in $\mathbb{R}^d$.

In [4], it was proven by using correlation inequalities, that in ϕ^4 models, g has an absolute upper bound (depending only on d) and in [3] it was argued that the value of g for ϕ^4 models should be dominated by its critical point value. The resulting picture of critical point dominance and its relation with renormalization group analysis is presented rather clearly in [11] (see especially Fig. 5 there) where

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