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Local Ward Identities and the Decay of Correlations in Ferromagnets

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Abstract. Using local Ward identities we prove a number of correlation inequalities for N-component, isotropically coupled, pair interacting ferromagnets; some for all $N \geqq 2$ and some for $N = 2, 3, 4$. These are used to prove a mass gap above the mean field temperature, for all $N \geqq 2$. For $N = 2, 3, 4$ we prove an upper bound on a critical exponent, and a lower bound on the susceptability which diverges as $m \to 0$.

1. Introduction

Recently, Dobrushin and Pecherski [1] announced some new results about the possible rates of clustering in equilibrium states of lattice systems with finite range interactions. One of the results is that if the clustering falls-off faster than a certain (dimension dependent) power of the separation, then it is exponential. In the above work, the clustering is expressed by a rather strong condition, which measures the independence of the statistical distribution of spins in any region from all the other spins which are further then a given distance away. Subsequently, in a work published in this issue, Simon [2] formulated, and proved for ferromagnetic Ising models, a new inequality which implies such a property for the two point correlation function. Thus, this inequality leads to an upper bound on the corresponding critical exponent. Furthermore, the inequality was used in [2] to provide upper bounds for the critical temperature of the mass gap. In fact, incorporating an improvement due to Lieb [3], one obtains a sequence of upper bounds, calculable by finite algorithms, which converge to the exact value. The derivation of the mass gap from the above mentioned inequality of [2] is related to its derivation from Griffith's third inequality [4], see [5, 6]. The latter is a particular case of the new inequality, in its improved version of [3].

The main purpose of this note is to prove inequalities similar to those of [2] for multicomponent, ferromagnetic, spin models with $O(N)$ symmetry. We use in the

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