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On the Families of Gibbs Semigroups

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Abstract. The families of Gibbs semigroups with generators from conveniently bounded monotonous families of self-adjoint operators are proved to be compact in the trace-norm topology.

1. Introduction

Let $\mathscr{H}$ be a separable Hilbert space and $\mathscr{C}_p(\mathscr{H})$ be the Banach space of compact operators on $\mathscr{H}$ with finite p-norm:

$$\|A\|_p = \left(\sum_{n=1}^{\infty} (\lambda_n(A))^p \right)^{1/p}, \quad 1 \leqq p < \infty .$$

Here $\{\lambda_n(A)\}_{n=1}^{\infty}$ are the singular values of the operator $A \in \mathscr{C}_p(\mathscr{H})$. Then one has: $\mathscr{C}_1(\mathscr{H})$ (trace-class) $\subset \mathscr{C}_2(\mathscr{H})$ (Hilbert-Schmidt operators) $\subset \ldots \subset \mathscr{C}_p(\mathscr{H}) \subset \ldots \subset \mathscr{C}_\infty(\mathscr{H})$ (compact operators) $\subset \mathscr{L}(\mathscr{H})$ (bounded operators) [1].

Strongly continuous one-parameter self-adjoint semigroups $\{Q(t)\}$, $Q: \mathbb{R}_+^1 \cup \{0\} \to \mathscr{L}(\mathscr{H})$, with the values in $\mathscr{C}_1(\mathscr{H})$ for $t \in \mathbb{R}_+^1$, arise naturally in quantum statistical mechanics. Usually they are called *Gibbs semigroups* [2, 3]. However, in some cases they are not to be self-adjoint (e.g. for $t \in \mathbb{C}_+$, the complex right half-plane $\{z \in \mathbb{C} : \operatorname{Re} z > 0\}$, or see Example 1). Hence, more relevant is the following

Definition 1. A strongly continuous semigroup $Q(t)$ in a separable Hilbert space is called a Gibbs semigroup if $Q(t): \mathbb{R}_+^1 \to \mathscr{C}_1(\mathscr{H})$.

Every strongly continuous semigroup $Q(t)$ is known (see e.g. [4, X.8] or [5, IX, § 1]) to be generated by a closed densely defined quasi-m-accretive operator T (semigroup generator) and has the form: $Q(t) = \exp(-tT)$.

Example 1. Quasi-m-accretive generators arise in perturbation theory of Gibbs semigroups [3, 6–8]. Let T be a positive generator of Gibbs semigroup. Suppose that U is a symmetric T-bounded operator with T-bound $b < 1$, i.e.

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