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Wall and Boundary Free Energies

III. Correlation Decay and Vector Spin Systems

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Abstract. The asymptotic free energy of planar walls and boundaries is analyzed for scalar and vector spin systems. Under the hypothesis of correlation decay, various alternative definitions are found to be equivalent in the thermodynamic limit and independent of the "associated" walls. Furthermore, a torus, or box having periodic boundary conditions, is shown to have no boundary or surface free energy. For vector spin systems with n-component spins, existence of the thermodynamic limit is shown for $n=2$ and "positive" interactions.

9. Introduction

The problem of the existence, uniqueness and properties of the free energy associated with a wall or boundary has been considered in two previous papers [1,2], to be referred[1] to below as I and II. In this paper, we consider (i) the boundary free energy in the one phase region, in particular, under the assumption of a uniform correlation decay law; (ii) the boundary free energy for a system of *vector* spins.

It has already been noted in I [see hueristic counterexample: in Sect. 2.7] that the free energy per unit area, $f_\times(K, W, \tilde{W}, \Omega)$, of a planar wall cut in the domain Ω is generally dependent on the associated wall potentials, $\tilde{W}$, imposed on the original boundaries of Ω, whenever the thermodynamic state is on a first order phase boundary so that the system may exhibit two-phase behavior. However, in a one-phase region, which for ferromagnetic spin systems is generally characterized by $T>T_c$ or $h\neq 0$, we expect the free energy associated with a wall to be independent of the associated walls. It is generally expected that, in the one-phase region, the correlations between two sets of spins, s_A and s_B, will vanish as the separation,

1 This paper is written as a direct continuation of Part II [Caginalp, G., Fisher, M.E.: Commun. Math. Phys. **65**, 247–280 (1979)]. Accordingly the numbering of sections and equations runs straight on from Part II

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