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The Equations of Wilson's Renormalization Group and Analytic Renormalization

II. Solution of Wilson's Equations

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Abstract. Wilson's renormalization group equations are introduced and investigated in the framework of perturbation theory with respect to the deviation of the renormalization exponent from its bifurcation value. An exact solution of these equations is constructed using analytic renormalization of the projection hamiltonians introduced in Paper I.

1. Introduction

This paper is a continuation of [1] hereafter referred to as I. Reference to equations or statements in I is made as follows: Eq. (I.3.1), Proposition I.5.1, etc. We directly pass here to solving the renormalization group (RG) equations in the framework of perturbation theory. The set-up of the paper is the following. First, in Sect. 2 we define the chain of Wilson's equations and find a set of bifurcation values of the RG parameter a. In Sect. 3 we consider the analytic continuation in a of a class of projection hamiltonians introduced in I. In Sect. 4 theorems on the analytic renormalization of projection hamiltonians of a special form are given and in Sect. 5 the RG transformation for these hamiltonians is described. These results enable us to construct in Sect. 6 a solution of the chain of Wilson's equations. In Sect. 7 and Appendix some auxiliary results are proved.

2. Wilson's Equations

These equations arise when one seeks nontrivial fixed points of the renormalization transformation near the bifurcation points. Before giving precise definitions, we want to explain their meaning. We expect that, as usual in many problems of nonlinear analysis, for certain values of the parameter a, a new branch of non-Gaussian solutions bifurcates from the branch of Gaussian fixed points of the RG (see Proposition I.1.1). Typically, this new branch is unique, however several branches may arise in degenerate cases. One can try to construct non-Gaussian solutions on this new branch as power series in the deviation of the

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