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The Relation between Relativistic Strings and Maxwell Fields of Rank 2

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Abstract. The local correspondence between sufficiently smooth relativistic string motions and Maxwell fields of rank 2 is proved.

In two recent papers [1, 2] Kastrup studied an electromagnetic field which is connected in a special way with a relativistic string [3, 4].

He started from the following string motion $x(\tau,\sigma)$:

$$\begin{aligned} &x^0=\tau, \quad x^1=A(\sigma-\tfrac{\pi}{2})\cos\omega\tau, \quad x^2=A(\sigma-\tfrac{\pi}{2})\sin\omega\tau, \\ &x^3=0, \quad 0\leqq\sigma\leqq\pi, \quad A\omega=\frac{2}{\pi}, \end{aligned} \tag{1}$$

which may be interpreted geometrically as a 2-dimensional surface $\Sigma^{(2)}$ in Minkowski space. Then Kastrup introduced Plücker's coordinates on $\Sigma^{(2)}$:

$$v^{\mu\nu}:=\dot{x}^\mu x'^\nu-\dot{x}^\nu x'^\mu, \quad \dot{x}^\mu:=\frac{\partial x^\mu}{\partial\tau}, \quad x'^\mu:=\frac{\partial x^\mu}{\partial\sigma}, \tag{2}$$

and constructed an electromagnetic field $F^{\mu\nu}(x)$ which is proportional on $\Sigma^{(2)}$ to $v^{\mu\nu}$:

$$F^{\mu\nu}(x(\tau,\sigma))=\lambda v^{\mu\nu}(\tau,\sigma), \quad \lambda=\text{const}. \tag{3}$$

The nontrivial part of the problem is to ensure that the field $F^{\mu\nu}(x)$ is a solution of the homogeneous Maxwell's equations:

$$\partial_\mu {}^*F^{\mu\nu}=0, \quad {}^*F_{\mu\nu}=\tfrac{1}{2}\varepsilon_{\mu\nu\varrho\sigma}F^{\varrho\sigma}, \quad \partial_\mu=\frac{\partial}{\partial x^\mu}.$$

[We use the metric $(+1,\,-1,\,-1,\,-1)$; $\hbar=c=1$.]

The remaining (inhomogeneous) Maxwell's equations may be taken as a definition of the current $j^\nu:=\partial_\mu F^{\mu\nu}$!

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