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Minkowski Space-Time is Locally Extendible

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Abstract. An example of a real analytic local extension of Minkowski space-time is given in this note. This local extension is not across points of the b-boundary since Minkowski space-time has an empty b-boundary. Furthermore, this local extension is not across points of the causal boundary. The example indicates that the concept of local inextendibility may be less useful than originally envisioned.

1. Extensions

The subject of extensions is playing an important role in General Relativity. In particular, it is crucial to the study of singularities (cf. [1, 2]). In studying singularities, it is necessary to distinguish between true singularities which are irremovable and apparent singularities which arise merely because the given space-time is a proper subset of a larger space-time.

An *extension* of a space-time (M,g) is a space-time (M',g') and an isometry $f:M\to M'$ which is onto a proper open subset of M'. Since space-times are connected, the set $f(M)$ must have a nonempty boundary $\mathrm{Bd}\, f(M) = $ closure $(f(M)) - f(M)$. Simple arguments based on the fact that $\mathrm{Bd}\, f(M) \neq \emptyset$, show that an extendible space-time cannot be timelike, null or spacelike geodesically complete. A space-time with no extensions is said to be *inextendible*.

A *local extension* of the space-time (M,g) is an open subset U of M which has noncompact closure $\bar{U}$ in M and an extension (U',g') of $(U,g|U)$ with isometry f mapping U into U' such that the image $f(U)$ has compact closure in U' (cf. [2, p. 59]). If (M,g), (U',g') and $f:U\to U'$ are all real analytic, then the local extension is said to be *analytic*.

Since closed subsets of a compact Hausdorff space are compact, a compact space-time is neither extendible nor locally extendible. Thus questions of extend-

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