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# On the Lagrangian Theory of Anti-Self-Dual Fields in Four-Dimensional Euclidean Space

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**Abstract.** We show that a certain four-dimensional field theory has powerful structures in common with the two-dimensional $0(1,3)$ non-linear $\sigma$-model.

## I. Introduction

By now, many non-equivalent two-dimensional relativistic and non-relativistic *integrable* field theories have been identified. Both their classical aspects, e.g. soliton solutions, action-angle variables, phase shifts and their quantum theoretical aspects, e.g. conservation laws, spectrum, scattering matrix, vacuum expectation values are being intensively studied. However, a complete and explicit classification of these theories is still lacking.

To the author's knowledge, not a single four-dimensional integrable field theoretical model – different from the free field – has been identified be it relativistic or not. Leaving aside the completeness requirement for the set of conserved charges which enters into the definition of integrable systems, actually, the very existence of a non-free four-dimensional continuum field theory with an infinite number of conserved charges has not been established, yet. On account of theorems due to Aks [1] on the one hand and to Coleman and Mandula [2] on the other hand it seems very unlikely that reasonable four-dimensional non-trivial *relativistic* theories exist which possess an infinite number of conserved *local* charges.

In this communication we show that a certain classical four-dimensional local Euclidean field theory admits an infinite number of continuity equations involving non-local expressions of the field variables and an infinite number of corresponding non-local symmetries. The theory in question was introduced into the literature by Yang [3]. It is a local translation-invariant though not manifestly SO(4)-invariant Lagrangian theory. Its extremal classical configurations are closely related to the anti-self-dual SU(2) Yang-Mills gauge fields, whence the name: Lagrangian theory of anti-self-dual fields.

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