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Space-Time Symmetries in Gauge Theories

P. Forgács and N. S. Manton

Laboratoire de Physique Théorique de l'Ecole Normale Supérieure,* F-75231 Paris Cedex 05, France

Abstract. A general definition of symmetries of gauge fields is proposed and a method developed for constructing symmetric fields for an arbitrary gauge group. Scalar fields occur naturally in the formalism and the pure gauge theory reduces to a Higgs model in lower dimensions.

1. Introduction

In the study of solitons (instantons, monopoles, vortices) in non-abelian gauge theories, space-time symmetry plays an important role. In the case of monopoles, the only solutions which are explicitly known are those which generalize the 't Hooft–Polyakov monopole [1], which is spherically symmetric. The BPST single instanton solution of the SU(2) self-duality equations [2] exhibits four-dimensional rotational symmetry, and Witten's more general multi-instanton solutions still have three-dimensional rotational symmetry [3].

The technique used in all these cases is to find an ansatz for the gauge fields and Higgs fields, possessing the desired symmetry, to insert into the field equations. In the case of 3-D spherical symmetry, for example, such an ansatz is required to be invariant under the combined rotation generated by $\mathscr{J}_i$,

$$\mathscr{J}_i = \mathscr{L}_i + \mathscr{S}_i + \mathscr{T}_i, \tag{1.1}$$

where $\mathscr{L}_i$ is the orbital angular momentum, $\mathscr{S}_i$ the spin, and $\mathscr{T}_i$ a generator of an SO(3) subalgebra of the gauge group [4]. We see that this technique involves embedding the symmetry group into the gauge group, which would also be the case for more general symmetries.

In this article we completely reconsider the question of symmetries in gauge theories. A set of symmetries is a group of motions of the underlying space-time manifold, with which we associate transformations of the fields defined on it. For a scalar field, this transformation would be just a shift of the argument. A gauge field, A_μ, possesses this group of symmetries if the appropriate transformation only has the effect of a gauge transformation on A_μ, hence leaving physical, gauge invariant quantities the same. We formulate this mathematically,

* Laboratoire propre du C.N.R.S. associé à l'Ecole Normale Supérieure et à l'Université de Paris Sud

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