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# Phase Transitions for Two-Dimensional Models with Isotropic Short-Range Interactions and Continuous Symmetries

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**Abstract.** Using the powerful method of reflection-positivity and chess-board estimates, we prove the existence of phase transition for certain class of isotropic short-range interactions with continuous symmetry, provided that the dimension of the lattice is at least two, and the temperature is low enough.

## 1. Introduction

There has been great progress recently in proving the existence of phase transitions for systems of statistical mechanics. The powerful methods of reflection-positivity and chess-board estimates were applied to both classical and quantum cases [1]. This made it possible to handle many anisotropic nearest-neighbor interactions [1] and to recover some results of Dyson and Kunz-Pfister for long-range isotropic interactions [2]. Further results of the authors of [2] are announced for Coulomb gases and quantum field theory.

The purpose of this paper is to show the possibility of applying the methods of [1, 2] to isotropic short-range interactions with continuous symmetry. For such interactions there exists a general result of [3] about absence of breakdown of continuous symmetry. So the one-point correlation functions must be the same in all phases. Indeed, in models we consider, the difference between phases is manifested by the difference between two-point correlation functions of these phases. The phases we will construct in Sect. 2 are translation invariant (after suitable change of coordinates). It does not contradict, however, results of [4, 5] about uniqueness of translation-invariant states for some $S^1$-invariant interactions, because we are dealing with an entirely different class of models. Nevertheless, several correlation inequalities of [4, 5] are also valid for our interactions (those of Ginibre type).

Finally, about the content of the paper. Section 2 contains the proof of the simplest variant of our results. This is done for the sake of clarity. The general result is formulated in Sect. 3. As the proof of it is essentially the same, it is omitted. The paragraph is concluded by some discussions.

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