Commun. Math. Phys. 68, 187–194 (1979)

Communications in Mathematical Physics

Chaotic Behavior in the Hénon Mapping

Frederick R. Marotto

Department of Mathematics, Drexel University, Philadelphia, PA 19104, USA

Abstract. In a previous work Hénon investigated a two-dimensional difference equation which was motivated by a hydrodynamical system of Lorenz. Numerically solving this equation indicated for certain parameter values the existence of a "strange attractor", i.e., a region in the plane which attracts bounded solutions and in which solutions wander erratically. In the present work it is shown that this behavior is related to the mathematical concept of "chaos". Using general methods previously developed, it is proven analytically that for some parameter values the mapping has a transversal homoclinic orbit, which implies the existence of the chaotic behavior observed by Hénon.

1. Introduction: The Hénon Mapping

In a recent work Hénon [2] investigated the dynamics of the mapping of the plane into itself defined by the difference equation:

$$\begin{aligned} x_{k+1} &= y_k + 1 - a x_k^2, \\ y_{k+1} &= b x_k, \end{aligned} \tag{1}$$

where $a, b \in \mathbb{R}$. Numerically solving (1) for a variety of initial values, he found this system to exhibit a very complex type of behavior. In particular for certain values of a and b Hénon found the existence of a "strange attractor" in (x_k, y_k) phase-space, that is, a region in the plane which attracts bounded solutions from outside under iteration of (1), and in which trajectories of (1) exhibit essentially random behavior. The implications of such behavior are significant. Once a strange attractor is observed, very unpredictable behavior of solutions will result. This is due to a lack of global stability of any solution, and more importantly, an extreme sensitivity to initial conditions. Ruelle and Takens [8] have suggested that such behavior is related to turbulence in the flow of fluids.

The principal motivation for consideration of (1) was an analysis conducted by Lorenz [4] upon a system of partial differential equations describing finite am-

0010-3616/79/0068/0187/$01.60