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A Remark on Dobrushin's Uniqueness Theorem

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Ten years ago, Dobrushin [1] proved a beautiful result showing that under suitable hypotheses, a statistical mechanical lattice system interaction has a unique equilibrium state. In particular, there is no long range order, etc.; see [6, 7] for related material, Israel [4] for analyticity results and Gross [3] for falloff of correlations.

There does not appear to have been systematic attempts to obtain very good estimates on precisely when Dobrushin's hypotheses hold, except for certain spin $\frac{1}{2}$ models [6, 4]. Our purpose here is to note that with one simple device one can obtain extremely good estimates which are fairly close to optimal.

Let Ω be a fixed compact space (single spin configuration space), $d\mu_0$ a probability measure on Ω and for each $\alpha \in Z^\nu$, Ω_α a copy of Ω. For X a finite subset of $\mathbb{Z}^\nu$, let $\Omega^X = \mathop{\mathsf{X}}_{\alpha \in X} \Omega_\alpha$. An interaction is an assignment of a continuous function, $\Phi(X)$, on Ω^X to each finite $X \subset \mathbb{Z}^\nu$. While it is not necessary for Dobrushin's theorem, it is convient notationally to suppose Φ translation covariant in the obvious sense.

Let $\mathscr{E} = \mathop{\mathsf{X}}_{\alpha \neq 0} \Omega_\alpha$ be the set of "external fields" to $\alpha = 0$. Given $s \in \Omega_0$, $t \in \mathscr{E}$, Φ with $\sum_{0 \in X} \|\Phi(X)\|_\infty < \infty$, we define $H(s|t)$ on Ω_0 by

$$H(s|t) = \sum_{0 \in X} \Phi(X)(s,t)$$

and for any t, the probability measure $\nu_t = e^{-H(\cdot|t)} d\mu_0(\cdot)/\mathbb{Z}_t$ with $\mathbb{Z}_t = \int e^{-H(s|t)} d\mu_0(s)$. Let

$$\varrho_i = \sup\{\tfrac{1}{2}\|\nu_t - \nu_{t'}\| \mid t_k = t'_k \text{ for } k \neq i\}, \tag{1}$$

where the norm on measures is the total variation norm:

$$\|\nu\| = \sup\{|\nu(f)| \mid f \in C(\Omega);\ \|f\|_\infty = 1\}.$$

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