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On the Hénon Transformation

James H. Curry*

Advanced Study Program, National Center for Atmospheric Research**, Boulder, Colorado 80307, and Department of Meteorology, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Abstract. In [4] Hénon studied a transformation which maps the plane into itself and appears to have an attractor with locally the structure of a Cantor set cross an interval. By making use of the characteristic exponent, frequency spectrum, and a theorem of Smale, our numerical experiments provide evidence for the existence of two distinct strange attractors for some parameter values, an exponential rate of mixing for the parameter values studied by Hénon, and an argument that there is a Cantor set in the trapping region of Hénon.

1. Introduction

In [4] Hénon, motivated by computer studies of the Lorenz system performed by Pomeau, studied a transformation which maps the plane into itself. Hénon was able to prove, among other things, that the transformation which he considered was the most general quadratic map which carries the plane into itself and has constant Jacobian determinant. Then in a remarkable sequence of computer graphics he gave strong numerical evidence that the transformation he studied has a strange attractor whose local structure is the product of a one-dimensional manifold by a Cantor set, at least in the neighborhood of one of the stationary solutions.

Further, Hénon was able to show for the specific parameter values which he considered that there exist a compact set M, called a "trapping region", which is carried into itself by the action of the transformation. Subsequently, Feit in [2] has generalized the above result by giving a characterization of the compact set of nondivergent points for Hénon's transformation – a point in [2] is called nondivergent provided its forward orbit under the action of the transformation is bounded.

In [2] characteristic exponents were also computed for a substantial set of parameter values for the Hénon map. If the characteristic exponent is less than

* On leave from the Department of Mathematics, University of Colorado, Boulder, Colorado 80309, USA

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