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Uniqueness and the Global Markov Property for Euclidean Fields. The Case of Trigonometric Interactions

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Abstract. We prove the global Markov property for the Euclidean measure given by weak trigonometric interactions. To obtain this result we first prove a uniqueness theorem concerning the set of regular Gibbs measures corresponding to a given interaction.

1. Introduction

The interest in extending the theory of Markov processes to the case where the time parameter is more than one dimensional, the so called Markov fields, arises in several domains, in particular in connection with problems of statistical mechanics, information theory and quantum field theory. The two main problems in the theory of Markov processes, the existence and uniqueness of the limit distribution and the question of the global Markov property are also main problems in the theory of Markov fields. For instance the question of the existence of a limit distribution corresponds in statistical mechanics and quantum field theory to the question of the existence of a Gibbs measure, while the uniqueness corresponds to the question whether this Gibbs measure defines a pure physical phase, non uniqueness indicating phase transitions.

The global Markov property on the other hand implies in statistical mechanics the existence of the transfer operator, and in quantum field theory it implies that the field theory is canonical and the zero time fields generate a maximal abelian subalgebra.

In this work we first prove that in the case of the Euclidean fields on $\mathbb{R}^2$ with weak trigonometric interaction one has uniqueness and also the global Markov property.

Concerning uniqueness corresponding results were proved in certain lattice and continuous models of statistical mechanics first by Dobrushin [13] (for further discussions see [14] and [15]).

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