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The Positive Action Conjecture and Asymptotically Euclidean Metrics in Quantum Gravity

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Abstract. The Positive Action conjecture requires that the action of any asymptotically Euclidean 4-dimensional Riemannian metric be positive, vanishing if and only if the space is flat. Because any Ricci flat, asymptotically Euclidean metric has zero action and is local extremum of the action which is a local minimum at flat space, the conjecture requires that there are no Ricci flat asymptotically Euclidean metrics other than flat space, which would establish that flat space is the only local minimum. We prove this for metrics on R^4 and a large class of more complicated topologies and for self-dual metrics. We show that if $R^\mu_\mu \geqq 0$ there are no bound states of the Dirac equation and discuss the relevance to possible baryon non-conserving processes mediated by gravitational instantons. We conclude that these are forbidden in the lowest stationary phase approximation. We give a detailed discussion of instantons invariant under an $SU(2)$ or $SO(3)$ isometry group. We find all regular solutions, none of which is asymptotically Euclidean and all of which possess a further Killing vector. In an appendix we construct an approximate self-dual metric on $K3$ – the only simply connected compact manifold which admits a self-dual metric.

1. Introduction

It has been expected for some time [1, 2, 3] that matter should be unstable when quantum gravity is taken into account. That is one expects gravity at some non-perturbative level to give rise to baryon and lepton number non-conservation. This is most clearly indicated in the external field theory computations of black hole evaporation [2]. One would like to compute processes of this sort using a fully quantized theory of gravity. The version of Quantum Gravity which seems most appropriate to us is the functional approach.

In the functional integral formulation of flat space quantum field theory physical quantities are expressed formally as functional integrals of the form

$$Z = \int_C d[\varphi] O[\varphi] \exp iI[\varphi] \,. \tag{1.1}$$

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