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Two-Cluster Scattering of N Charged Particles

Volker Enss

Department of Theoretical Physics, University of Bielefeld, D-4800 Bielefeld 1, Federal Republic of Germany*, and
Institut des Hautes Etudes Scientifiques, F-91440 Bures-sur-Yvette, France

Abstract. *We define geometrically two-cluster scattering states by their asymptotic space-time behavior.* We show that these subspaces coincide with the ranges of the two-cluster wave operators, or modified wave operators if both clusters are charged. In particular this proves asymptotic completeness and absence of a singular continuous spectrum of the Hamiltonian below the lowest three-body threshold.

Introduction

It is common belief that two-cluster scattering and two-particle (=potential-)scattering are closely related. Asymptotically it should not matter whether two particles or two bounded subsystems move apart from each other and become free. Indeed Combes [2] could prove asymptotic completeness below the three-body threshold where the asymptotic breakup into more than two clusters is energetically forbidden. Simon simplified [14] Combes' proof and extended it [15] to include more potentials. Simon's treatment allowed charged particles but at least one of the clusters had to be neutral[1]. Our method includes the case where both clusters are charged (which requires the use of modified wave operators), also more general short range interactions which may be velocity dependent are included.

To avoid the energy restriction we introduce two subspaces ($\mathbf{2}_{\text{in}}, \mathbf{2}_{\text{out}}$) describing the incoming or outgoing two-cluster scattering states. We characterize them "geometrically", i.e. by their behavior in space and time. In these states the particles within each of the clusters "stay together" asymptotically in the past or future whereas the clusters separate (we give the precise definition below). This is a *natural extension of Ruelle's geometric characterization of bound states and* scattering states [13].

We show that $\mathbf{2}_{\text{in/out}}$ coincides with the direct sum of the ranges of the corresponding two-cluster (modified) wave operators. Using compactness arguments we can decompose any state from $\mathbf{2}_{\text{out}}$ at a sufficiently late time into a finite

* Present address
1 See "Note Added in Proof"