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Generic Instability of Rotating Relativistic Stars

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Abstract. All rotating perfect fluid configurations having two-parameter equations of state are shown to be dynamically unstable to nonaxisymmetric perturbations in the framework of general relativity. Perturbations of an equilibrium fluid are described by means of a Lagrangian displacement, and an action for the linearized field equations is obtained, in terms of which the symplectic product and canonical energy of the system can be expressed. Previous criteria governing stability were based on the sign of the canonical energy, but this functional fails to be invariant under the gauge freedom associated with a class of trivial Lagrangian displacements, whose existence was first pointed out by Schutz and Sorkin [12]. In order to regain a stability criterion, one must eliminate the trivials, and this is accomplished by restricting consideration to a class of "canonical" displacements, orthogonal to the trivials with respect to the symplectic product. There nevertheless remain perturbations having angular dependence $e^{im\phi}$ (ϕ the azimuthal angle) which, for sufficiently large m, make the canonical energy negative; consequently, even slowly rotating stars are unstable to short wavelength perturbations. To show strict instability, it is necessary to assume that time-dependent nonaxisymmetric perturbations radiate energy to null infinity. As a byproduct of the work, the relativistic generalization of Ertel's theorem (conservation of vorticity in constant entropy surfaces) is obtained and shown to be Noether-related to the symmetry associated with the trivial displacements.

I. Introduction

In the introduction to their 1970 paper on cosmological singularities, Hawking and Penrose [1] noted that gravity is an "essentially unstable" force. For small concentrations of mass, the instability is masked by enormously larger short range forces. But when the density of matter is sufficiently large or its mass sufficiently great, gravity becomes dominant and collapse inevitable. From this instability to collapse arises the theoretical expectation of black holes; and the strongest observational argument in their favor is provided by the associated upper limit on

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