Commun. math. Phys. 62, 83—96 (1978)

Communications in Mathematical Physics

Resonances in Stark Effect and Perturbation Theory

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Abstract. It is proved that the action of a weak electric field shifts the eigenvalues of the Hydrogen atom into resonances of the Stark effect, uniquely determined by the perturbation series through the Borel method.

This is obtained by combining the Balslev-Combes technique of analytic dilatations with Simon's results on anharmonic oscillators.

I. Introduction

The Stark effect on a Hydrogen-like atom is described by the Hamiltonian operator:

$$H(F) = -\Delta - Z/r + 2Fx_3 \tag{1.1}$$

acting on $L^2(R^3)$. Here $2F>0$ is the uniform electric field directed along the x_3 axis, Z the atomic number, and $r=(x_1^2+x_2^2+x_3^2)^{1/2}$.

As is well known, the Schrödinger operator (1.1) is a non-positive singular problem in perturbation theory of linear operators, and the spectrum of $H(F)$ is (absolutely) continuous in $(-\infty, +\infty)$, while the spectrum of the unperturbed operator $H(0)$, the Hydrogen atom, is discrete along $(-\infty, 0)$. The spectral theory of this problem is then treated within the framework of asymptotic perturbation theory [12] based upon strong convergence of resolvents as $F \to 0$.

In addition, an alternative technique for dealing specifically with this kind of problems has been very recently developed by Avron and Herbst [1] and by Veselič and Weidmann [24].

It involves considering $-\Delta + 2Fx_3$ as the unperturbed operator and Z/r as the perturbation and yields the absolute continuity of the spectrum along $(-\infty, +\infty)$ and the existence of the wave operators.

By means of the strong convergence of resolvents Riddell [17] generalizing earlier results of Titchmarsh [22], proved that the spectrum of (1.1) along $(-\infty, 0)$

* Partially supported by G.N.F.M., C.N.R.

** Partially supported by I.N.F.N., Sezione di Bologna

0010-3616/78/0062/0083/$02.80