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On the Determination of Cauchy Surfaces from Intrinsic Properties*

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Abstract. We consider the problem of determining from intrinsic properties whether or not a given spacelike surface is a Cauchy surface. We present three results relevant to this question. First, we derive necessary and sufficient conditions for a compact surface to be a Cauchy surface in a spacetime which admits one. Second, we show that for a non-compact surface it is impossible to determine whether or not it is a Cauchy surface without imposing some restriction on the entire spacetime. Third, we derive conditions for an asymptotically flat surface to be a Cauchy surface by imposing the global condition that it be imbedded in a weakly asymptotically simple and empty spacetime.

I. Introduction

In the initial value formulation of general relativity, one starts with initial data on a surface S and evolves that data to produce a maximal 4-dimensional region, $D(S)$, the Cauchy development of S (see [1] and pp. 244–255 of [2]). One goal of the initial value problem is to find sufficient intrinsic conditions on the surface S and its data which will guarantee that its Cauchy development $D(S)$ is an inextendible spacetime. (The surfaces A and B in Figure 1 have developments which are extendible.) Such sufficient conditions are not known.

If $D(S)$ is extendible, then there always exists a maximal extension M which is an inextendible spacetime. (This is easily proved using Zorn's lemma [3].) Then, one might ask whether there is some other surface S' whose development is all of M; i.e. $D(S')=M$. We prove a theorem which shows that if S satisfies certain intrinsic conditions but is not a Cauchy surface for M, then M has no Cauchy surface at all (i.e. M is not globally hyperbolic). Stated another way, these intrinsic conditions on S are sufficient to guarantee that if M is globally hyperbolic then S is

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