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On a Class of Polynomials Connected with the Korteweg-deVries Equation*

M. Adler[1] and J. Moser[2]

[1] University of Wisconsin, Madison, Wisconsin 53706, USA
[2] New York University, New York, New York 10012, USA

Abstract. A new and simpler construction of the family of rational solutions of the Korteweg-deVries equation is given. This construction is related to a factorization of the Sturm-Liouville operators into first order operators and a new deformation problem for the latter. In the final section the spectral representation for the corresponding complex potentials is discussed.

1.

In [1] special classes of solutions of the Korteweg-deVries equation

$$u_t = 3uu_x - \tfrac{1}{2}u_{xxx} = X_2 u \tag{1.1}$$

were studied, in particular all those $u = u(x,t)$ which are rational functions of x for each value of t. It turns out that these solutions are rational functions of t as well and of very special structure. In this paper we give a new construction of these solutions with emphasis on their algebraic properties.

To describe the family of rational solutions of (1.1), one does well to introduce the sequence of associated Korteweg-deVries equations

$$u_t = X_k(u), \quad k = 1,2,\ldots, \tag{1.2}$$

which are related by

$$X_k = \frac{\partial}{\partial x}\frac{\delta H_k}{\partial u}$$

to the sequence of conserved quantities

$$H_k = \int P_k(u, u', \ldots)\, dx$$

associated with (1.1). These X_k can be recursively defined by

$$X_{k+1}(u) = \left(u\frac{\partial}{\partial x} + \frac{\partial}{\partial x}u - \frac{1}{2}\left(\frac{\partial}{\partial x}\right)^3\right)\frac{\delta H_k}{\partial u}$$

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