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Comments on Mielnik's Generalized (Non Linear) Quantum Mechanics*

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Abstract. We discuss a model of non linear quantum mechanics in which the wave equation satisfies the homogeneity condition (2.1). It is argued that in this model the set of (mixed) states is a simplex.

I. The Setting

The term "state" of a system is customarily used in quantum physics for a statistical ensemble of (equally prepared) samples of the system. Since such ensembles may always be mixed the set $\mathscr{S}$ of states is a convex set, i.e. with $\omega_1, \omega_2 \in \mathscr{S}$ and $0 < \lambda < 1$

$$\omega = \lambda\omega_1 + (1-\lambda)\omega_2 \tag{1.1}$$

is again a state, namely the mixture of ω_1 and ω_2 with weights λ and $(1-\lambda)$. The extremal points of the convex set $\mathscr{S}$ (i.e. those states ω which cannot be represented as a mixture of others) are the pure states. We denote their set by $\mathscr{E}$. Since we shall be concerned here with the simplest possible generalization of ordinary quantum mechanics it suffices to consider the case where $\mathscr{S}$ is "atomic" i.e. where every state ω is a countable convex combination of pure states[1]

$$\omega = \sum \lambda_i \Phi_i, \quad \Phi_i \in \mathscr{E}, \quad \lambda_i > 0, \quad \sum \lambda_i = 1 . \tag{1.2}$$

Still $\mathscr{E}$ will not determine $\mathscr{S}$ because in general different mixtures of pure states may result in ensembles which are indistinguishable by any measurement. The limitations in observability introduce an equivalence relation (denoted by $\sim$) in

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1 More generally one might take $\mathscr{E}$ to be a measure space and replace (1.2) by

$$\omega = \int_{\mathscr{E}} \Phi d\mu(\Phi) \tag{1.3}$$

with μ a (positive, normalized) measure on $\mathscr{E}$

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