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Planar Diagrams

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Abstract. We investigate the planar approximation to field theory through the limit of a large internal symmetry group. This yields an alternative and powerful method to count planar diagrams. Results are presented for cubic and quartic vertices, some of which appear to be new. Quantum mechanics treated in this approximation is shown to be equivalent to a free Fermi gas system.

1. Introduction

We present some investigations of the planar approximation to field theory calculated through a limit of a large internal symmetry. Part of the motivation for this work lies in the hope that it might ultimately provide a mean of performing reliable computations in the large coupling phase of non-abelian gauge fields in four dimensions. In addition there are some indications that such topological expansions are related to the dual string models [1]. To support these hopes we may quote the significant simplifications occuring in the large N-limit for the linear or non-linear σ-models which indeed allow to discriminate the phases of broken and unbroken symmetry (even in two dimensions where the symmetry is never broken). On the other hand one has 't Hooft's solution to two-dimensional QCD in this same limit [2]. These promising features suggest to pursue this line of reasoning and develop some new techniques.

A first part of this paper is devoted to preliminary combinatorial aspects [3]. Some of these have already been discussed by Koplik, Neveu and Nussinov [4]. The method that we have used for this "zero-dimensional" field theory, in which every propagator is set equal to unity, is not of combinatorial nature and hopefully allows for extension to genuine calculations of Green functions in a real field theory. This enabled us to solve a few counting problems the solution of which does not seem to be known.

In Section 5, we compute explicitly the contribution of all the planar Feynman diagrams to the ground state energy of a one dimensional gx^4-anharmonic

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