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Scalar Quantum Electrodynamics on the Lattice as Classical Statistical Mechanics

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Abstract. Wilson's lattice approximation allows us to apply classical statistical mechanics ideas to the study of Scalar Quantum Electrodynamics. Our main tools are Griffiths-Kelly-Sherman inequalities, the transfer matrix formalism and exponential bounds. Our main result is the existence of the infinite volume limit for every value of the coupling parameters.

1. Introduction and Outline

In 1974 Wilson [18] suggested the interesting possibility of performing the Yang-Mills [19] step of promoting a global symmetry to a local one *after* introducing a lattice ultraviolet cutoff on the globally invariant theory. This procedure not only gave a very convenient gauge invariant prescription for the ultraviolet regularization of gauge theories, but also, due to the use of a Euclidean space-time lattice, as compared to the space lattice-continuous time of the alternative Hamiltonian approach [10], opened the possibility of using classical statistical mechanics methods for the study of quantum gauge fields in the spirit of the general program of Quantum Field Theory as Classical Statistical Mechanics [5, 9, 13]. An analysis of the structure of the statistical mechanical models thus introduced and a preliminary exploration by mean field techniques of their properties was given in [1]. An extensive rigorous discussion, in the strong coupling regime, of the thermodynamic limit and of the existence of a mass gap was given in [14, 15]. Important upper bounds holding for any value of the coupling constant for the pure $U(1)$ model were given in [6]; a very promising step towards the full understanding of the critical behavior of the same model is in the analysis of [7] based on a mathematically rigorous examination of the role of Polyakov's instantons [16]: this method suggests that there is no mass generation at sufficiently low coupling for $d \geqq 4$, as opposed to the $d=3$ case.

This paper is devoted to the study of Wilson's version of Scalar Quantum Electrodynamics on the lattice briefly described in Section 2. In Section 3 we discuss

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