Commun. math. Phys. 56, 213—224 (1977)

Communications in
Mathematical
Physics
© by Springer-Verlag 1977

Stability Properties of Equilibrium States

Rudolf Haag and Ewa Trych-Pohlmeyer*

II. Institut für Theoretische Physik der Universität Hamburg, D-2000 Hamburg, Federal Republic of Germany

Abstract. The significance of stability of an equilibrium state under local perturbations of the dynamics (as defined in [1]) and the different degree of stability with respect to extended perturbations of states at phase transition points are discussed. The general conclusions are tested and illustrated in the example of the free Bose gas. A more transparent proof of the relation between local stability and the Kubo-Martin-Schwinger relation is given.

I. Introduction

It was argued in [1] that thermodynamic equilibrium states of an infinitely extended medium are distinguished among (possibly other) stationary states by a certain stability with respect to small changes of the dynamical law. In fact, this stability should be considered as the defining property of an equilibrium state. Specifically we consider the quantum physics of an infinitely extended system [1]. The system is described by the algebra $\mathfrak{A}$ of its quasi local observables, the states by expectation functionals on $\mathfrak{A}$ and the dynamics by the 1-parameter automorphism group α_t. [2]

The conceptual definition of stability is then the following: Consider a small change of the dynamical law to the automorphism group α_t^h which results from α_t by the "addition of a perturbation Hamiltonian h". Then if ω is a stationary state with respect to α_t it is called stable under this perturbation if there exists a state ω^h, stationary with respect to α_t^h, which is close to ω. In particular, in [1] we took h to be an element of $\mathfrak{A}$, which means physically that we consider essentially local, bounded perturbations. If λ is a coupling constant which we let tend to zero ultimately then the stability requirement is that $\|\omega^{\lambda h}-\omega\| \to 0$ as $\lambda \to 0$ for all such perturbations i.e.

---

* Present address: Institut für Theoretische Physik der Universität Heidelberg, D-6900 Heidelberg, Federal Republic of Germany

1 This definition of equilibrium states may also be used in classical statistical mechanics. See [4, 5]

2 Notation: $\omega(A)$ denotes the expectation value of the element $A \in \mathfrak{A}$ in the state ω. $\alpha_t(A)$ is the time translated element A