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A Note on the Inverse Scattering Problem in Quantum Field Theory

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Abstract. We study the set of local fields ϕ describing the dynamics of a scalar, massless particle. It turns out that these fields are relatively local to the free, massless, scalar field A if the massless particle does not interact. This leads to a simple algebraic characterisation of interacting fields in the above framework.

1. Problems and Results

An old problem in quantum field theory is to characterize all local fields leading to a given S-matrix [1], [2; Chapter 4.6], [3; Chapter 19.5]. Our interest in this question arose from a desire to have a local criterion distinguishing field theories with interaction from non-interacting ones. For that purpose it would be sufficient to know all fields leading to a trivial S-matrix. However, even this simpler problem has not yet been solved.

In view of this situation we found it worth while putting the following analysis on record although it applies only to a rather special case. We consider in this paper the model of a scalar, massless particle which does not interact. It turns out that there each interpolating field ϕ for the (trivial) S-matrix is an element of the Borchers class [1] of the free, massless field A. All these elements are explicitly known (see [4] and the Appendix) and this solves the inverse scattering problem for this special model. Moreover, because the free, massless field commutes with itself at timelike separations, one obtains a simple algebraic characterisation of interacting fields in the above framework: a local field ϕ, describing the dynamics of a scalar, massless particle leads to a non-trivial S-matrix if and only if the commutator $[\phi(x), \phi(y)]$ does not vanish at timelike distances $(x-y)$.

Our argument is based on the following reasoning: if the asymptotic fields coincide, $\phi^{\mathrm{in}}=\phi^{\mathrm{out}}=A$, they have the same TCP-operator as the interpolating field ϕ; consequently, ϕ and A are weakly local with respect to each other [1]. Moreover, by Huyghens' principle [5] the commutator between $\phi(x)$ and $A(y)$ vanishes at timelike distances $(x-y)$. Hence if B is any element of the Borchers class of A the support of the vacuum expectation value

$$K(x)=(\Omega, [B(0), \phi(x)]\Omega) \tag{1}$$