

Wall and Boundary Free Energies

I. Ferromagnetic Scalar Spin Systems

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Abstract. The existence of wall or boundary free energies is discussed generally and analyzed explicitly for general lattice systems with scalar (real-valued) spin variables. For systems with ferromagnetic (positive) spin interaction potentials, K , in the bulk and W , for the walls, correlation inequalities and appropriate stability and tempering conditions are used to establish the existence and uniqueness of the limiting free energy per unit area, $f_\times(K, W)$, of an infinite planar wall.

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