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Isolated Maximal Surfaces in Spacetime

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Abstract. Maximal surfaces and their implications for the ambient spacetime are studied. Our methods exploit the interplay between contact of the volume functional and energy conditions. Essentially, we find that in closed universes, maximal surfaces are unique; they maximize volume; and they yield future and past singularities.

I. Introduction

The global properties of manifolds, in particular the types of extremal surfaces which they admit, have attracted much interest. In Riemannian manifolds, minimal surfaces—soap films and their generalizations to higher dimensions—have been the subject of numerous investigations [1]. The analogous surfaces in spacetimes, manifolds with Lorentz metric, are maximal surfaces. In asymptotically flat spaces, frequently a family of maximal surfaces folicates spacetime [2]. Such a foliation then gives a geometrically preferred time function. Maximal surfaces are also physically important for such questions as the analysis of the dynamics of the gravitational field or the analysis of the n-body problem in a gravitational field [3]. Abundant as they are in asymptotically flat spacetimes, maximal surfaces occur but rarely in spatially closed universes. Not surprisingly, their physical interpretation is also quite different. Namely, they typically describe the "turn around" epoch which separates the expansion from the recontraction phase. Today there are no theorems about the maximum expansion phase comparable in generality to the singularity theorems [5], which describe the universe's collapse. Here we initiate the study of the "turn around" epoch.

We consider three dimensional spacelike surfaces in spacetimes and the critical points of their volume functional. In Section II, we give the variational formulas for the first, second and higher variations of the volume away from a maximal

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