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Greens Functions, Hamiltonians and Modular Automorphisms

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Abstract. We demonstrate, under circumstances that allow the construction of a "thermodynamic" hamiltonian, that Gibbs equilibrium states ω are modular states in the Tomita-Takesaki sense. The thermodynamic Greens functions G are connected to these modular states, and the associated group of modular automorphisms σ, by the identification

$$G(A, B; t) = \omega(A\sigma_t(B))$$

(A and B are observables) whenever the thermodynamic Hamiltonian is self-adjoint and defines a derivation of the algebra of observables in a certain sense. Our results apply to a class of interacting quantum gases at small fugacity and Bose gases with repulsive interactions at all fugacities $z < 1$.

1. Introduction

Although the time-development of thermodynamic systems in quantum statistical mechanics is barely understood, some progress has been made on the questions of existence and characterization of equilibrium. The existence problem has been tackled by establishing that the limits

$$\tau_t(A) = \lim_{\Lambda \to \infty} e^{iH_\Lambda t} A e^{-iH_\Lambda t}$$

of the time development of finite systems Λ exist in some suitable sense. (H_Λ represents the Hamiltonian of a finite system Λ, A a fixed observable, and the limit indicates the thermodynamic limit of an idealized infinite system.) Two types of limit, i.e. types of convergence, have been distinguished.

In the best cases (free fermi gas, "short range" spin systems) the above limits exist for all A in a C^*-algebra $\mathfrak{A}$ of quasi-local observables and $t \to \tau_t$ defines a strongly continuous one-parameter group of automorphisms of $\mathfrak{A}$ [1–3]. Under these circumstances one can also show that all Gibbs equilibrium states (limits