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Bounded Perturbations of Dynamics

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Abstract. If α and α' are one-parameter automorphism groups of a von Neumann algebra $\mathfrak{M}$, α' is said to be a bounded perturbation of α if $\|\alpha'_t - \alpha_t\| \to 0$ as $t \to 0$. We give a complete characterization of the bounded perturbations α' of α. In particular, we show that if α can be implemented by a strongly continuous one-parameter group with self-adjoint generator (Hamiltonian) H, then α' can be implemented in the same way and the corresponding Hamiltonian H' can be chosen to be of the form $H' = VHV^{-1} + h$, where V is a unitary of $\mathfrak{M}$ and $h = h^* \in \mathfrak{M}$.

1. Introduction

The equilibrium states of an infinite system in quantum statistical mechanics are believed to be characterized by the KMS-condition [1, 2]. Recently Haag et al. [3] and Kastler [4] have taken the important step of deriving this condition from postulates allowing a direct physical interpretation. The novel ingredient of their approach is the claim that equilibrium states should be stable under small local perturbations of the dynamics. Their result has stimulated us to take a fresh look at the concept of perturbations of dynamics and to ask what is the natural class of gentle perturbations of dynamics within the framework of the algebraic approach to quantum theory.

If one thinks in terms of quantum mechanical perturbation theory, one would perturb dynamics by replacing the total Hamiltonian H of the system by

$$H' = H + \lambda h \tag{1.1}$$

where h is a bounded observable and λ a small coupling constant. When dealing with infinite quantum systems, the total Hamiltonian has a direct physical significance only in the case of quantum field theory, so that it is important to rewrite the *Ansatz* (1.1) in a form which makes no explicit reference to the total Hamiltonian. The dynamics must be thought of as being described instead by a con-

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