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# Remarks on the Wilson-Zimmermann Expansion and Some Properties of the $m$-Point Distribution*

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**Abstract.** This paper contains a few simple remarks on a paper by S. Schlieder and E. Seiler. For the special class of local fields treated by these authors we arrive at the same necessary condition for the existence of the Wilson-Zimmermann expansion (considered both as an operator expansion and as an expansion in bilinear forms) of the product of $n$ real scalar fields under the assumption that the singularities occurring as $x_j \to x_{j+1}$; $j=1,2,\ldots,n-1$, do not influence each other as long as these limits are simultaneously taken.

## 1. Introduction

In this paper the connection between the Wilson-Zimmermann expansion of the product of $n$ local fields and some properties of the $m$-point distribution will be discussed. This discussion is restricted to the special class of Wightman fields discussed by E. Seiler and S. Schlieder [1] (see also Ref. [2]).

If the Wilson-Zimmermann expansion exists then the singularities arising in the $2n$-point distributions for the set $\{x_j \to x_{j+1}; j=n+1, n+2, \ldots, 2n-1\}$ must control the singularities of the $m$-point distribution ($m > 2n$) for the set $\{x_j \to x_{j+1}; j=k, k+1, \ldots, k+n-2$; with $k \leqq m-n+1\}$. In this paper simple generalizations of one or two lemmas and of the theorem in Ref. [1] will be given. Let us start by considering the Wilson-Zimmermann expansion of the product of $n$ real scalar fields. If $\xi' = (\xi'_1, \ldots, \xi'_{n-1})$ and $x = \frac{1}{n}\sum_{i=1}^{n} x_i$ where $\xi'_j = (x_{j+1} - x_j)/2\varrho$, $j=1,2,\ldots,n-1$, and $\varrho > 0$, the Wilson-Zimmermann asymptotic expansion can be written [3, 4] in the following manner

$$\begin{aligned}&(\Phi, A(x+\varrho\alpha_1)\ldots A(x+\varrho\alpha_n)\Psi)\\&= \sum_{j=1}^{k} f_j(\varrho)(\Phi, C_j(x,\xi')\Psi) + R_{k+1}(\Phi, \Psi; x, \xi')\end{aligned} \tag{1.1}$$

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