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On the Point Spectrum of the Schrödinger Operators of Multiparticle Systems

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Abstract. It is shown that the point spectrum of the n-particle Schrödinger operators in the center-of-mass frame is finite for shortrange and dilation analytic potentials.

1. Let C be a subset of $(1\ldots n)$ and H^C, the Schrödinger operator of the system C in its center-of-mass frame. We omit the index C for $C=(1\ldots n)$. The general structure of the spectrum of H for, say, the Kato potentials is given by the following statement[1].

Theorem 1 ([1–3]). (i) *The essential spectrum of the operator H coincides with the half-line $[\mu, \infty)$ where*[2]

$$\mu=\min\left\{\lambda \mid \lambda\in \bigcup_{\{C_i\}} \sum_{\substack{C_i \subsetneqq (1\ldots n)\\ \cup C_i=(1\ldots n)}} \sigma_P(\mathrm{H}^{C_i})\right\}.$$

(ii) *The set of accumulation points of $\sigma_P(H)$ is included in*

$$\left\{\lambda \mid \lambda\in \bigcup_{\{C_i\}} \sum_{\substack{C_i \subsetneqq (1\ldots n)\\ \cup C_i=(1\ldots n)}} \sigma_P(H^{C_i})\right\}.$$

The second statement was proved in the additional assumption of the dilation analyticity of the potentials (see p. 157).

2. Further and exhaustive information on the continuous spectrum of the Schrödinger operator of a multiparticle system has been obtained in scattering theory [4–7].

The next basic problem in a qualitative description of the point spectrum of the Schrödinger operator is to find classes of potentials for which the point

[1] In this work we use the following standard definitions: the point spectrum $\sigma_p(A)$ of an operator A is the set of all eigenvalues of finite multiplicities of A. The discrete spectrum $\sigma_d(A)$ of A is the set of all isolated eigenvalues of finite multiplicities.

[2] For $C=(k)$ we set $H^C=0$ and $\sigma_p(H^C)=\sigma(H^C)=\{0\}$. The union is taken over $\{C_i\}$ such that $\sigma_p(H^{C_i})\neq \emptyset\,\forall i$.